Exercise 22

(a) How long will it take an investment to double in value if the interest rate is 6% compounded continuously?

(b) What is the equivalent annual interest rate?

Solution

Part (a)

If the interest is compounded continuously, the value of an investment after t years is

$$A(t) = A_0 e^{rt},$$

where A_0 is the initial investment and r is the interest rate. To find how long it will take an investment to double with 6% interest, set $r = 0.06$ and $A(t) = 2A_0$ and solve the equation for t.

$$2A_0 = A_0 e^{0.06t}$$

$$2 = e^{0.06t}$$

$$\ln 2 = \ln e^{0.06t}$$

$$\ln 2 = (0.06t)\ln e$$

$$t = \frac{\ln 2}{0.06} \approx 11.5525 \text{ years}$$

Part (b)

In order to double an initial investment in this time with annual compound interest, solve the following equation for r.

$$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt} \quad \rightarrow \quad 2A_0 = A_0\left(1 + \frac{r}{1}\right)^{1\left(\frac{\ln 2}{0.06}\right)}$$

$$2 = (1 + r)^{(\ln 2)/0.06}$$

$$\ln 2 = \ln(1 + r)^{(\ln 2)/0.06}$$

$$\ln 2 = \left(\frac{\ln 2}{0.06}\right)\ln(1 + r)$$

$$0.06 = \ln(1 + r)$$

$$e^{0.06} = 1 + r$$

Solve for r.

$$r = e^{0.06} - 1 \approx 0.0618365$$

Therefore, to double the money in 11.5525 years, the annual interest rate would have to be about 6.18%.